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ON THE VISUAL COMPLEXITY OF BUILT AND NATURAL LANDSCAPES

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Abstract

In this study, we analyze some critical points of the application of the box-counting method to the evaluation of the fractal dimensions of the natural and built landscapes. A brief theoretical discussion of the eventual drawbacks of the method is supported by experimental results of two box-counting programs applied to classical fractals. The optimized version of the algorithm, based on the results of computations for the classical fractal images, is proposed and employed for the evaluation of the complexity level of the chosen historical buildings and surrounding environment in the well-known case of Amasya city. The hypothesis of the relationship between the visual complexity of built and natural settings is analyzed for the Amasya case and for two historical Brazilian cities.

Keywords: Computational Fractal Analysis; Fractal Dimension; Box-Counting Method; Natural Landscapes; Historic Buildings.

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1. INTRODUCTION

The first studies on the fractal geometry of natural environments have appeared in the 80s, and one of the ramifications of these works, important for our research, is the analysis of the visual complexity of natural landscapes.[1–4] In general terms, it consists of the extraction of silhouettes from the landscape images and the evaluation of the fractal dimension of the resulted simplified forms. A general conclusion reached was that different landscape types are characterized by different fractal dimensions.[1,4,5]

The fractal analysis of the built environment started in the 90s, first involving the evaluations of the complexity of urban planes at the entire city and street scale.[6–10] At a smaller scale of buildings and group of buildings, the scale relevant for our study, the first results were presented by Bechhoefer and Bovill[11] and by Bovill,[12] where, in particular, the box-counting method for calculation of the fractal dimension was applied to characterize the visual complexity of the considered historical buildings. Since then, a similar technique was applied to the analysis of other historical buildings and settings.[13–16]

One of the important points of fractal studies on the urban environment is the investigation of the relation between the visual complexity of the urban scenes and surrounding natural scenes. First, the hypothesis of such a relationship was put forward in the works of Bovill and collaborators,[11,12] and it also was partially tested there. The hypothesis attracted scholars and was tested in different cases, but the results obtained are inconclusive and controversial.[12,16–18] The focus of our study is to review one of the known test cases and verify the validity of the hypothesis in the case of two Brazilian cities.

It appears that all the approaches used so far can be divided into two groups: empirical studies, and factual analysis. In general terms, the former suggests for a chosen group of participants a list of artificially created scenes of the urban and natural landscapes, and asks to choose the scenes that match each other better and worse. The results of the answers are measured on a chosen scale for the studied characteristic (it can be roughness, complexity, similarity, etc.), so in this way all the results are transformed into numerical form and are subsequently subjected to statistical evaluation. If the pool of respondents is sufficiently large and the standard deviation is sufficiently small, then in some statistics one can derive conclusions on the validity and stability of the obtained results. Such approach was used in many studies.[1,18–20] In particular, in the work by Stamps[18] the hypothesis that the fractal dimension of the skyline matches the fractal dimension of the surrounding landscape was tested, and in the research of Hagrehall *et al.*[1] the human preference for natural landscapes with certain levels of fractal complexity was investigated.

Among the problematic points of the empirical method, one can mention the subjectivity and artificiality in the creation of the proposed scenes, the subjectivity in the choice of the numerical scale for measuring qualitative responses, the necessity of having a large and representative pool of the respondents, and the absence of a mathematical theory on the validity of the results, because the obtained distributions are usually not normal.

The second approach consists of the comparison of the geometric complexity of the actual urban and natural settings. Usually, the box-counting method is applied to available graphic material (photographs, images, maps, etc.) to evaluate the fractal dimension of the characteristics chosen for comparison.[11,12,15–17] A number of studies were concerned with the texture analysis of the images, including those representing the natural scenes.[21–25] In particular, in the last three articles, fractal analysis was applied to the binary images and the validity of this method for the classification of the image complexity was shown. A similar approach was employed in the study of urban settings by Bovill,[12] Lorenz,[17] Cooper and Oskrochi,[8] Cooper *et al.*[9] The details of this approach, as applied in our research, are specified below in Secs. 2–4, where the box-counting algorithm is used to calculate the fractal dimensions of the chosen historical buildings and the corresponding natural environments. Although some subjective elements are also pertinent to this approach (as it will be seen in details in Secs. 2 and 3), it has very attractive points including the strong mathematical basis in the case of theoretical fractals and the possibility to analyze the images of complex objects, such as real landscapes and constructions. The latter property is important to minimize the presence of artificial elements and make possible a more objective evaluation of the preferences of large groups of inhabitants.

The text is structured as follows. In Sec. 2, basic definitions of the fractal theory are recalled and some properties of the box-counting method are discussed. Section 3 is concerned with testing of the chosen software and discovering the properties important for image treatment. The program performance and essential properties are verified using the images of classical fractals, whose exact fractal dimension is known, and then the improved algorithm is applied to the known case of Amasya city to compare with the results of the previous studies.[12,16,17] The analysis and main results of the comparison between the visual complexity of the natural and built sites for the cases of two historical Brazilian cities, Ouro Preto and Pelotas, are presented in Sec. 4, followed by the concluding remarks in the final section.

2. BOX-COUNTING METHOD: THEORETICAL CONSIDERATIONS

The well-known Mandelbrot definition of a fractal, made in his fundamental essay, states that "a fractal is ... a set for which the Hausdorff–Besicovitch dimension strictly exceeds the topological dimension".[26] However later Mandelbrot regretted that "the definition is very general, which is desirable in mathematics. But in science its generality was to prove excessive: no only awkward, but genuinely inappropriate".[27] Besides, the Hausdorff dimension itself is a complex mathematical concept. So, like many other authors,[1,6,12,15,28] we appeal to a non-exact general description of a fractal as a geometrical structure with the following properties: it is irregular at any scale; it cannot be described with the required precision using traditional geometry; its non-traditional (fractal) dimension, defined in a proper way, is usually greater than its topological dimension.

The box-counting fractal dimension is one of the most popular measures of the geometric complexity in applied sciences due to its simple mathematical formulation and straightforward approximation through numerical algorithms for practically arbitrary forms. To avoid unnecessary generality and simplify considerations, let us consider the specific situation relevant for our study: a bounded figure located on a two-dimensional plane. Let N_r be the smallest number of squares with the side length r, which cover our figure; then the box-counting dimension is the following limit (if it exists): $D = -\lim_{r\to 0}\log N_r/\log r$. It is well-known that for some classes of fractals (such as self-similar fractals without or with sufficiently small overlapping), the box-counting dimension coincides with the Hausdorff dimension (for exact formulations see Falconer[29]).

In practice, one uses an approximate form of the definition, evaluating N_r for different fixed values of r and extrapolating the obtained results to the limit as r approaches 0. The most popular version of numerical approximation is as follows. First, the figure is covered by a rectangular mesh with square cells (boxes) having a length r, and the number N_r of the boxes, which contain at least one point of the figure, is counted. The same count is performed on a finer grid with a reduced linear size of boxes. For a simplicity of algorithm, frequently the reduction factor is chosen to be 2 (the power of 2 algorithm). This procedure of mesh refining and box counting is repeated until the finest possible mesh is reached. Then, the obtained values of the function N_r of the variable r are used to evaluate the fractal dimension of the figure. Frequently the least squares linear fitting for the curve $N_r(r)$ is applied and the slope of the found straight line is the approximation to the fractal dimension. Another option is to find the local slopes, defined as $(\log N_{r_2} - \log N_{r_1})/(\log r_2 - \log r_1)$ for two consecutive box lengths r_1 and r_2 (for the power of 2 algorithm it means $\log_2(N_{r_2}/N_{r_1})$), and consider the mean value of these slopes as the approximated fractal dimension.

Although very popular and simple in implementation, this algorithm has some drawbacks, which may affect the reliability of the results. Some issues are general for treatment of any image of a real structure. First of all, any real image has a finite resolution, therefore, according to exact definition, it can not be considered a fractal, but just its approximation. Actually, the same is true for any real (including natural and built) object, so even if the images and computers would have an infinite resolution, in reality the non-artificial objects have finite range of subdivisions and scales (at least we should stop reaching the atom scale) and do not satisfy the exact definition of a fractal. Another general problem is a multifractality: the majority of the natural and built forms have different levels of complexity at different space scales, and in this case it is difficult (or even impossible) to define a unique measure of complexity for the entire structure.

Various issues can arise due to imperfections and even deficiencies of images, which frequently are photographs of real objects and include imperfections of digital camera, shooting conditions and digital formats used for image storage. Some specific features can affect the results of computations. For example, the width of the lines can be non-uniform over the image and some lines can be larger than the smallest boxes. Also, the presence of some secondary objects, for example, cables, trees, high grass, etc., can contaminate the evaluation of the principal structures.

Finally, the power of 2 algorithm has additional issues. First, the original figure (or image) is not usually of a square or rectangular form, and, almost for sure, its size is not a power of 2, or a multiple of such a power. This requires some adjustment between the figure size and coverings on the boundaries of the figure, which can compromise the box counts especially at the large sizes. Second, since the powers of 2 decrease rapidly, one soon gets down to the precision of data, when the process should stop. So it may happen that only a few measurements will be obtained, insufficient to give a reliable result.

All these considerations lead to the following conclusions: a single fractal dimension of real objects generally does not exist; the primary interest is shifted to the local slopes (local fractal dimensions) connected with rugged forms at the chosen range of scales; the counts for the smallest and largest boxes are usually non-informative; different problems can be reflected in significant variations of approximations to the fractal dimension over different space scales. Thus, it is frequently much more important to analyze a sequence of the measurements related to different space windows, instead of obtaining a single number, which should represent an approximation to the supposed fractal dimension that probably does not even exist. Moreover, the main attention may be focused on the measures of local complexity, using, for example, the local slopes defined above. The behavior of the local slopes with respect to space scales can show the multifractal nature of a geometric form and the pattern of fractal measures, which can characterize a specific class of the structures. This can provide important information on both the variability and reliability of the results.

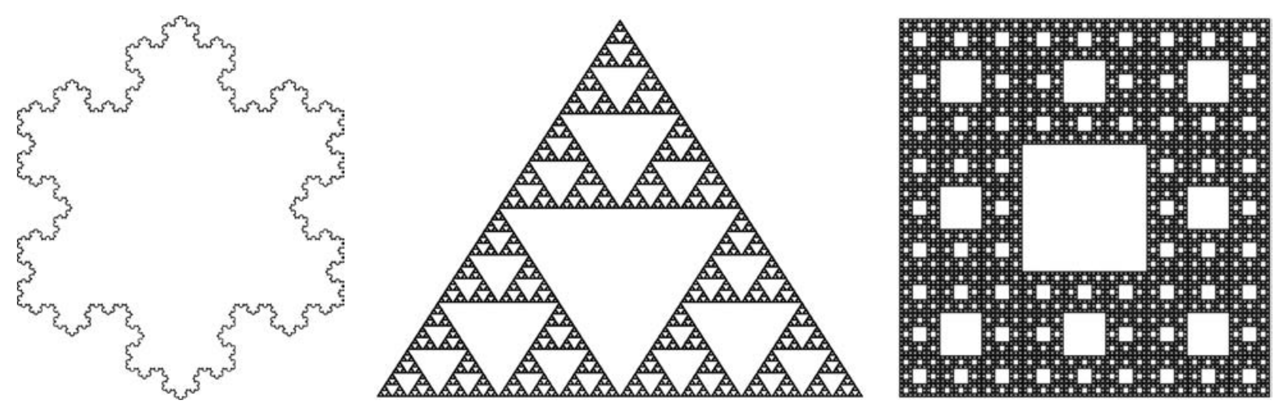
3. BOX-COUNTING PROGRAM: TESTS AND RELEVANT PROPERTIES

In this section, we present the results of Boxcount program by Moisy[30] and Fractalyse program by Frankhauser and Tannier[31] applied to the three classical fractals and to the known case of the evaluation of the visual complexity of the natural and built settings, which were studied independently by three groups of researchers. The results of both programs are practically the same, which is one of the forms of validation of the results.

3.1. Classical Fractals

First we test the program in the case of the three classical fractals with known theoretical fractal dimensions. We chose the fractals with different dimensions: the Koch snowflake of a lower complexity ($D = \log 4/\log 3 \approx 1.262$), Sierpinski triangle of a medium complexity ($D = \log 3/\log 2 \approx 1.585$) and Sierpinski carpet of a higher complexity ($D = \log 8/\log 3 \approx 1.893$). The images of these fractals are shown in Fig. 1.

In order to simulate the subsequent treatment of the images obtained with digital camera or prepared digitally using software, each of the fractals was



(a) The Koch snowflake. **(b)** The Sierpinski triangle. **(c)** The Sierpinski carpet.

Fig. 1 Three classical fractals.

Table 1 The Local Slopes of the Sierpinski Triangle.

Bs	1	2	3	4	5	6	7	8	9	10	11	12	13
N_r	301971	87287	26232	8356	2739	908	306	97	36	14	5	2	1
d	1.79	1.73	1.65	1.61	1.59	1.57	1.66	1.43	1.36	1.49	1.32	1.00	

Note: Bs – box size measured in pixels (the power of 2), N_r – the number of boxes, d – the local slopes.

generated by the Matlab program and then saved in jpeg format file of a medium resolution (300 dpi, 2400 × 1800 pixels). Due to computer limitations, the first two fractals were calculated with nine iterations of scale refinement, and the last one with six iterations. Then the images in the jpeg files were processed by the Matlab program, which transforms the color format to grayscale image, binarizes it, and counts the number of the square boxes covering the obtained silhouette (black/white simplification) of the original image. Of course, for the classical fractals the stages of forming a color-type image and then transferring it to the binarized form can be dropped. However, one of our goals at this stage of tests was to mimic the subsequent application of the program to the real color images, including some imperfections and distortions, which are partially imitated in the above process of creating jpeg-formatted files of the classical fractals with medium resolution.

The result of computations is the sequence of the box counts, corresponding to different coverings with the box length (measured in pixels) equal to a power of 2, and also the final approximation to the fractal dimension together with the standard deviation. Instead of the box numbers it is convenient to use the sequence of local slopes (defined in Sec. 2). For the Sierpinski triangle, such sequence is shown in Table 1 (hereinafter the results are ordered in the sequence of increasing box size).

It is evident that the covering values of the smallest and largest boxes have a weak (if any) relation to the theoretical fractal dimension. Indeed, if all the twelve numbers in Table 1 are used to calculate the mean value of the slope, then the result is not so bad (1.52), but it happens only due to accidental compensation of large and small wrong values in this specific case. The standard deviation from this mean value is 0.23, that is, roughly speaking, the probable approximations to the theoretical result have a great chance to be found in the entire interval [1.29, 1.75], which is very low level of precision. However, if we eliminate some problematic counts at the both ends of the space spectrum, then the results can be significantly improved. For example, using the box sizes from 4 to 8, we obtain much better result of 1.57 for the computational fractal dimension, and more importantly, the confidence level of this result is quite acceptable because the standard deviation in this case is 0.09. Even if we choose to drop only the two smallest sizes and the three largest sizes, we obtain the evaluation 1.58 for the fractal dimension and the standard deviation is still reasonable −0.12.

It is worth to note that the shown results are in line with those available in other sources (e.g., Buczkowski *et al.*,[32] Nonnenmacher *et al.*[33] and Wong *et al.*[34]). The main difference is that we highlight the behavior of the local slopes, which are more sensitive to space variations. Since the results in the above papers are presented in a more usual form of the evaluation through the curve fitting, we provide also the graph of the obtained box counts in the logarithmic scale ($\log N_r$ versus $-\log r$), along with the straight line of the least squares curve fitting in Fig. 2.

It is seen that the relationship between two curves is strong, which is also confirmed by the correlation coefficient of 0.9986 and relatively small

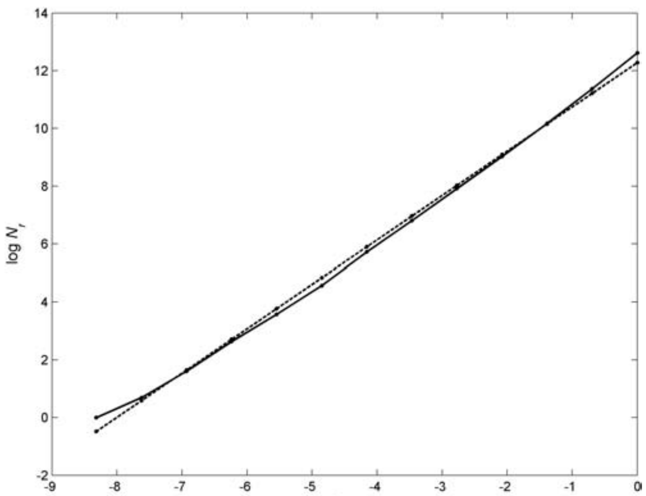


Fig. 2 The box counts graph and the fitting curve for the Sierpinski triangle.

standard deviation of 0.031. The slope of the straight line (1.53) is also a reasonable approximation to the theoretical fractal dimension (1.585). All these results resemble the corresponding evaluations in the above three papers. At the same time, the graph in Fig. 2 shows again that the maximum deviations between two curves occur at the endpoints of the spatial spectrum (albeit it is not highlighted at the same degree as for the measure by the local slopes). Therefore, just like in the case of the local slopes, we can obtain much better results by eliminating the extremal box sizes. For example, filtering out the two smallest and three largest sizes, we obtain the corresponding straight line slope of 1.60, the correlation coefficient of 0.9996 and the standard deviation of 0.0019.

The results for the Koch snowflake and Sierpinski carpet are quite similar and, for this reason, we provide here just a brief summary of them in order to show that the program keeps good performance for fractals with different levels of complexity. The main results for all the three classical fractals are assembled in Table 2 with averaging over all spatial scales shown in the first row, over the box sizes from 2 to 9 in the second row and from 4 to 8 in the third row (the corresponding values of the spatial scales s are shown in the first column). A lower level of approximation for the Sierpinski carpet can be attributed to the smaller number of iterations used to generate this fractal.

Table 2 **Fractal Dimensions of the Three Fractals.**

Spatial Range s	Snowflake		Triangle		Carpet	
	D	sd	D	sd	D	sd
1–12	1.31	0.47	1.51	0.22	1.67	0.54
2–9	1.32	0.13	1.58	0.12	1.78	0.12
4–8	1.28	0.09	1.57	0.09	1.79	0.11
Theoretical	1.262		1.585		1.893	

Note: D – fractal dimension (approximate and theoretical), sd – standard deviation.

It can be noted that the observed properties of the box-counting computation of the fractal dimension agree closely with the theoretical considerations presented in the previous section. Even for the figures, which represent the approximations to the well-defined single-fractal geometric forms, the level of approximation of the theoretical fractal dimension depends strongly on the box sizes used in the calculation of the mean value. Therefore, in order to optimize the evaluation of the fractal dimension, some filtration of the box counting results should be applied. Apparently, the use of both extremely small and large boxes just contaminates the results in the middle part of the space spectrum, which are very satisfactory. So it is reasonable to suggest that the one or two smallest sizes and two to three largest sizes should be eliminated from the final evaluation of the fractal dimension. In the following section it is shown that the tests involving images of natural and built settings confirm this proposition.

3.2. The Case of Amasya City

Bovill[12] tested the hypothesis on the relationship between the visual complexity of built and natural settings in the case of a specific historical area of the city of Amasya, Turkey, founded over 2000 years ago. In particular, he presented three images in this area — the Amasya hill, house elevation and urban plan (see Fig. 3) — and calculated their fractal dimensions using the box-counting method in order to substantiate his proposition.[12] Later the results obtained were reviewed independently by Lorenz,[17] and Vaughan and Ostwald.[16]

The results of calculations of the fractal dimension of three images are presented in Table 3: the first three columns reproduce the results obtained by Bovill,[12] Lorenz,[17] and Vaughan and Ostwald,[16] respectively, while the fourth column contains the results obtained by applying the algorithm described in Sec. 3.1 with filtration of the two smallest and three largest box sizes.

(a) Hill. **(b)** Elevation. **(c)** Urban plan.

Fig. 3 The Amasya images.

Table 3 Fractal Dimension of the Amasya Images According to Different Sources.

Element\Source	Bovill	Lorenz	Vaughan/ Ostwald	Algorithm with Filtering (Scales $s = 3-8$)		Algorithm without Filtering (Scales $s = 1-11$)	
	D	***D***	***D***	***D***	***sd***	***D***	***sd***
D_{hil}	1.57	1.36	1.50	1.69	0.14	1.45	0.38
$D_{elevation}$	1.72	1.55	1.51	1.77	0.14	1.58	0.44
$D_{urban\,plan}$	1.43	1.49	1.59	1.64	0.05	1.43	0.36

Note: D – fractal dimension, sd – standard deviation, s – the range of spatial scales used in the algorithm.

It is seen that for each of three evaluated elements, the results of the computations in four independent studies are different. It was suggested by Vaughan and Ostwald[16] that the distinctions among three first reports are due to inconsistency of computational implementations of the box-counting method: Bovill[12] made his calculations manually, Lorenz[17] used an early version of the software Benoit, while Vaughan with Ostwald[16] applied a refined version of the Benoit and also the Archimage software and presented the result averaged over the two programs. Indeed, this may be one of the sources of discrepancy in the results, as well as the differences in quality and computer form of the photographs used by the authors. However, we would like to draw attention to another feature that may be the main cause of the differences among the results. This feature is the presence of significant deviations in the distribution of the box counts with respect to the box size (shown in the parenthesis in the fourth column of Table 3) even for algorithm with filtration of the extremal sizes (spatial scales $s = 3-8$). In the fifth column we also present the results averaged over entire range of the used box sizes (without filtering the smallest and largest scales). It is seen, that when the most problematic coverings, corresponding to a few smallest and greatest scales, are eliminated, the level of deviations reduces significantly, and consequently the results in the fourth column are much more reliable than those in the fifth column. Hence, the range of box coverings involved and the method of processing these counts can influence strongly on the final result. This is the case of the considered Amasya images, as it is exemplified in the last two columns of Table 3. Besides, the characterization of the complexity of the natural and built landscapes through a single number may have a weak connection with actual complexity of objects due to multifractality, which requires a sequence of evaluations at the different space scales. In such cases it is helpful to use local measures like the local slopes.

Both Bovill[12] and Lorenz[17] take care of the local slopes and provide analysis of the differences in these slopes, including explanations in terms of the building details and structures. On the contrary, the analysis in the third work[16] is based only on the results of final approximations to the fractal dimensions, with no discussion about variability and reliability of the obtained results. In the course of their analysis of the Amasya case, Vaughan and Ostwald[16] arrive to the conclusion that the gap of 0.09 (defined as the difference between the highest and lowest fractal dimensions, i.e., the gap = 1.59−1.50) "suggests a significant difference in visual character". As one can see from Table 3, this statement is not applicable in the Amasya case because the standard deviations shown in the fifth column are so high (about four to five times greater than 0.09) that, based only on these numbers, one cannot draw a conclusion about the real value of the fractal dimensions with any reasonable precision. Even for the filtered results shown in the fourth column, such a conclusion is still impossible.

As for the Bovill hypothesis, it is hard to draw a conclusion with the base on the tests made for the city of Amasya, even in the case of the filtered results. There are too many indeterminations in all stages of evaluation, starting with imperfections of images and question of reliability of the hill image, and ending with relatively large deviations for the two fractal dimensions (the hill and elevations).

4. BOX-COUNTING METHOD APPLIED TO TWO BRAZILIAN CITIES

In this section, we apply the filtered algorithms of box-counting method for evaluation of the fractal complexity of two historical Brazilian cities — Ouro Preto and Pelotas. The city of Ouro Preto was founded at the end of the 17th century in the valley among the mountains of the state Minas Gerais, Brazil (it is located at the elevation of about 1.2 km). It is included in the UNESCO list of the World Heritage Sites due to its rich baroque architecture. Pelotas was founded at the beginning of the 19th century on a plain of a low elevation near the ocean in the state Rio Grande do Sul, Brazil. Its historical architecture is considered the national heritage due to important sites of the colonial and eclectic styles.

The two cities, Ouro Preto and Pelotas (OP and PE in notations in the following figures), were chosen for analysis of the visual complexity due to their quite different natural environments, which might influence the complexity of the city sites, and in this way might confirm (or not) the Bovill hypothesis. In what concern the built environment we restrict ourselves to street and building level scenes, because at this level the original historical constructions have suffered a smaller influence of the posterior central planning development of cities.

First we present in Figs. 4 and 5 two typical images of natural surroundings for each of the cities (all the presented images are already transformed to gray scale form as it was done in the program).

Each image in Figs. 4 and 5 was transformed to its binary form, according to the intensity values at each pixel, in order to extract the silhouettes of the presented objects. Examples of the black-white images representing textural complexity are presented in Fig. 6: the first image corresponds to

(a) Ouro Preto surroundings 1 (OPN1).

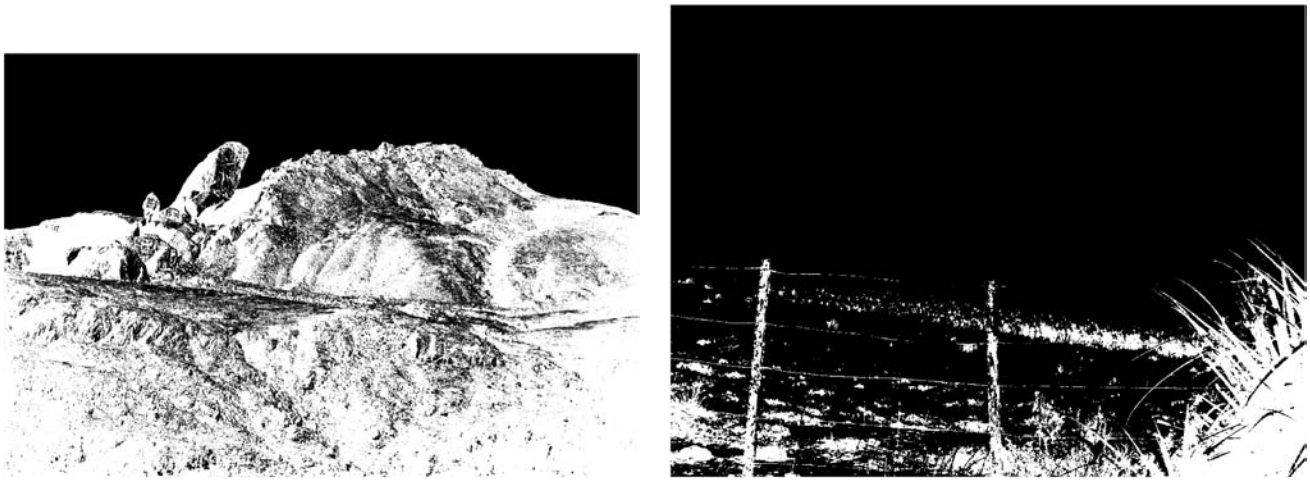
(b) Ouro Preto surroundings 2 (OPN2).

Fig. 4 The Ouro Preto surrounding environment.

(a) Pelotas surroundings 1 (PEN1).

(b) Pelotas surroundings 2 (PEN2).

Fig. 5 The Pelotas surrounding environment.



(a) Binary image of OPN1. **(b)** Binary image of PEN1.

Fig. 6 Black/white representations of the surrounding environment.

the Ouro Preto environment photo, and the second to the Pelotas environment.

Then the box-counting program was applied to compute the distribution of the local slopes along the space scales. The local slopes of the four images computed within the chosen space window (the scales remaining after filtering out the smallest and largest boxes) are shown in Fig. 7. Additionally, the approximated values of the fractal dimensions/standard deviations are: 1.92/0.07 for OPN1, 1.95/0.07 for OPN2, 1.73/0.09 for PEN1, and 1.82/0.08 for PEN2.

We do not focus here on specific values of the local slopes or on their mean value (albeit it is seen that the natural surroundings of Ouro Preto have higher values of the fractal dimension), but instead

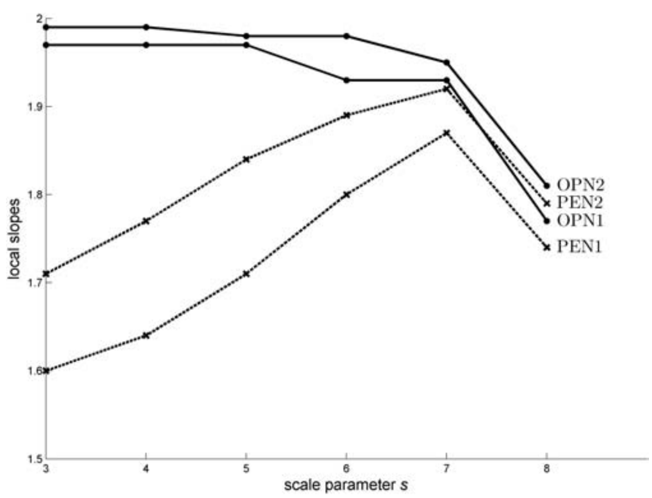


Fig. 7 The local slopes of the natural landscapes.

we draw attention to another characteristic — distribution of the slopes along the scale (box size) axis. It is a noticeable feature of the Ouro Preto distributions that the slopes keep almost the same value over all the range of scales and decrease only for very large box sizes. Differently, the Pelotas distributions have notably smaller values at the smallest scales and reach the maximum at the larger scales.

It is interesting to verify if the same features are pertinent to the corresponding architectural sites. In Figs. 8 and 9 we present the characteristic street scenes in the historical parts of the cities (two for each city) and the two corresponding samples of binary images are shown in Fig. 10.

The same box-counting algorithm provides the results of the fractal analysis shown in Fig. 11. The corresponding approximated values of the fractal dimensions/standard deviations are: 1.87/0.02 for OPB1, 1.86/0.02 for OPB2, 1.70/0.08 for PEB1, and 1.72/0.07 for PEB2.

Thus, both fractal dimensions for Pelotas are smaller than those of Ouro Preto, but it is much more important to compare the scale dynamics of the local slopes presented in the graphs. First, in Fig. 9, both fractal curves for Pelotas evaluations lies below the curves for Ouro Preto, indicating a less degree of complexity in the Pelotas scenes, which agrees with the relation between the visual complexities of the corresponding natural landscapes. Further, there exists a strong similarity between the curves of slopes for historical buildings and natural surroundings in each of the locations. At the same time, the spatial distributions of the

(a) Ouro Preto historical buildings 1 (OPB1).

(b) Ouro Preto historical buildings 2 (OPB2).

Fig. 8 The Ouro Preto historical urban scenes.

(a) Pelotas historical buildings 1 (PEB1).

(b) Pelotas historical buildings 2 (PEB2).

Fig. 9 The Pelotas historical urban scenes.

(a) Binary image of OPB1.

(b) Binary image of PEB1.

Fig. 10 Black/white representations of the urban scenes.

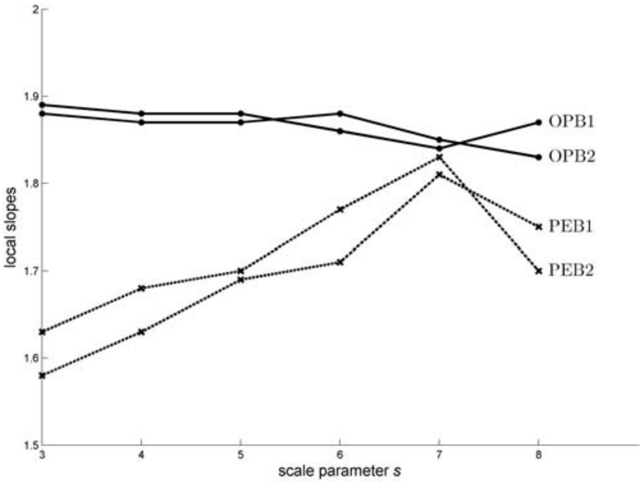


Fig. 11 The local slopes of the built landscapes.

slopes are visibly different if we compare the characteristics of two cities. These results suggest that in the studied cases the Bovill hypothesis on relationship between the visual complexity of built and natural settings is valid, at least qualitatively, without specifying the quantitative level of agreement.

5. CONCLUSIONS

In this study we have analyzed the hypothesis on the relationship between the visual complexity of built and natural settings. The geometric complexity was measured by the fractal dimension of buildings and environmental silhouettes. The important issues of the application of the box-counting method were discussed and some relevant properties were illustrated in the evaluation of the fractal dimensions of three classical fractals. The optimized algorithm was employed for the analysis of the Amasya case and it was shown that the results obtained cannot be used to derive a sound conclusion on the visual complexity of the three presented images due to large standard deviations. In the case of two Brazilian cities, Ouro Preto and Pelotas, the same algorithm revealed a strong connection between the features of the local slopes for the historical groups of buildings and the surrounding natural environments. It was shown that the considered built and natural scenes have a notable similarity in distribution of fractal complexity with respect to spatial scale. Since the standard deviations of the fractal measures were sufficiently small, we can conclude that the hypothesis on the relationship between the visual complexity of built and natural landscapes is confirmed for the considered Brazilian settings.

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